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Technical Note

Oscillatory double-diffusive convection in a porous enclosure due to opposing heat and mass fluxes on the vertical walls

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1. Introduction

Double-diffusive convection, which occurs because of temperature and concentration difference under gravity, is observed in many fields of disciplines, for example, electrochemistry, geophysics and so on [1–4]. Various authors have theoretically and numerically studied the double-diffusive convection in a fluid-saturated porous enclosure [5–13], giving analytical solutions in the tall cavity and numerical solutions under constant heat and mass fluxes. When the two fluxes have an opposite effect in buoyancy, we found that the numerical calculations gave oscillatory solutions [8,9]. The competition between heat and mass transfer with different diffusivities plays an important role to generate oscillations, which occur even at low Rayleigh numbers. In our previous report [8], we showed the oscillation only at a few discrete cases with related parameters. Therefore, it is not yet clear what ranges in parametric values are necessary to make convection oscillatory, and how the nature of oscillation varies with the related three parameters: Rayleigh numbers, Lewis numbers and buoyancy ratios. In this paper we have evaluated the oscillation range numerically by extending the values of the three parameters, and continuously varying those values. Oscillatory double-diffusive convection occurring without inertia term in a porous enclosure will shed light on further understanding of this kind of competitive and cooperative work by two forces, temperature and concentration differences.

2. Problem statements

The geometry used in the mathematical model is given in Fig. 1. We consider a two-dimensional vertical enclosure filled with a homogeneous fluid-saturated porous medium of aspect ratio A. The top and bottom walls are insulated. Constant heat flux Λ_T and mass flux Λ_c are prescribed through the vertical walls. The momentum conservation in the Darcy regime with Boussinesq approximation is used with the following equations:

$$\mathbf{u} = -\nabla P - R_c(\theta - N\phi)\mathbf{e}_y. \quad (1)$$

The equation of continuity:

$$\nabla \cdot \mathbf{u} = 0. \quad (2)$$

The equations for the mass and thermal energy conservation:

$$\varepsilon\frac{\partial\theta}{\partial t} + \mathbf{u}\cdot\nabla\theta = \nabla^2\theta \quad (3)$$

and

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Nomenclature

A	aspect ratio (dimensionless)
D	solute diffusivity ($m^2\ s^{-1}$)
f	non-dimensional frequency (dimensionless)
g	acceleration of gravity ($m\ s^{-2}$)
$2h$	enclosure width (m)
$2H$	enclosure height (m)
k	permeability (m^2)
Le	Lewis number (dimensionless)
N	buoyancy ratio (dimensionless)
Nu	Nusselt number (dimensionless)
P	pressure (dimensionless)
R_c	Rayleigh–Darcy number (dimensionless)
S	power spectrum intensity (dimensionless)
t	non-dimensional time (dimensionless)
$\mathbf{u}$	non-dimensional velocity vector = (u, v) (dimensionless)
x	non-dimensional horizontal coordinate (dimensionless)
y	non-dimensional vertical coordinate (dimensionless)

Greek symbols

α	coefficient of thermal expansion (K^{-1})
β	coefficient of concentration expansion ($m^3\ mol^{-1}$)
ϵ	porosity ratio (dimensionless)
ϕ	non-dimensional temperature
κ	thermal diffusivity ($m^2\ s^{-1}$)
Λ_c	horizontal concentration gradient prescribed on the side wall ($mol\ m^{-4}$)
Λ_T	horizontal temperature gradient prescribed on the side wall ($K\ m^{-1}$)
ν	kinematic viscosity ($m^2\ s^{-1}$)
θ	non-dimensional concentration (dimensionless)
σ	heat capacity ratio (dimensionless)

$$\sigma \frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = Le \nabla^2 \phi. \qquad (4)$$

The boundary conditions:

$$\frac{\partial \theta}{\partial x} = -1, \quad \frac{\partial \phi}{\partial x} = -1, \quad u = 0 \quad \text{and} \quad \frac{\partial v}{\partial x} = 0 \ \text{at} \ |x| = 1 \qquad (5)$$

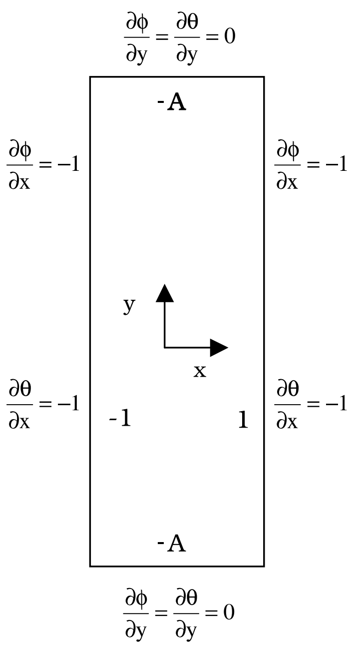

Fig. 1. The geometry of the porous enclosure.

and

$$\frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial \phi}{\partial y} = 0, \quad v = 0 \quad \text{and} \quad \frac{\partial u}{\partial y} = 0 \ \text{at} \ |y| = A. \qquad (6)$$

The initial conditions:

$$\theta = 0, \ \phi = 0 \ \text{and} \ \mathbf{u} = 0 \ \text{at} \ t = 0. \qquad (7)$$

The dimensionless parameters are defined as follows:

$$A = \frac{H}{h}, \quad Le = \frac{\kappa}{D}, \quad R_c = \frac{kg\beta\Lambda_c h^2}{\nu D} \quad \text{and} \ N = \frac{\alpha\Lambda_T}{\beta\Lambda_c}. \qquad (8)$$

Governing equations are solved numerically with the boundary and initial conditions by the finite difference method. The governing equations and the boundary conditions are discretized over a network of 62×302 grids in uniform spacing. No grid point is set on the physical boundaries ($|x| = 1$ and $|y| = A$). The first and end grid points are put at a distance of half a grid space away from the boundaries. Boundary conditions at the walls are given on these points. The numerical scheme used here is second-order accurate in space and first-order accurate in time. The matrices are solved under the given boundary conditions by the conjugate gradient method. For more details, see [8].

3. Results and discussion

When the heat flux is applied to the system in the opposite direction from the mass flux, the convective flow will be promoted in the same direction. If the two fluxes are in the same direction, the convective flow by the thermal flux is disturbed from that by the mass flux, resulting in complex convection. All the calculations in this paper are concerned with the latter case. The aspect ratio A is one of the key parameters to have the oscillating solution in numerical calculations under these conditions; the oscillation does not take place in the case of $A = 1$ when R_c is less than 200. In this paper, only the results obtained at $A = 5$ are shown, though computa-

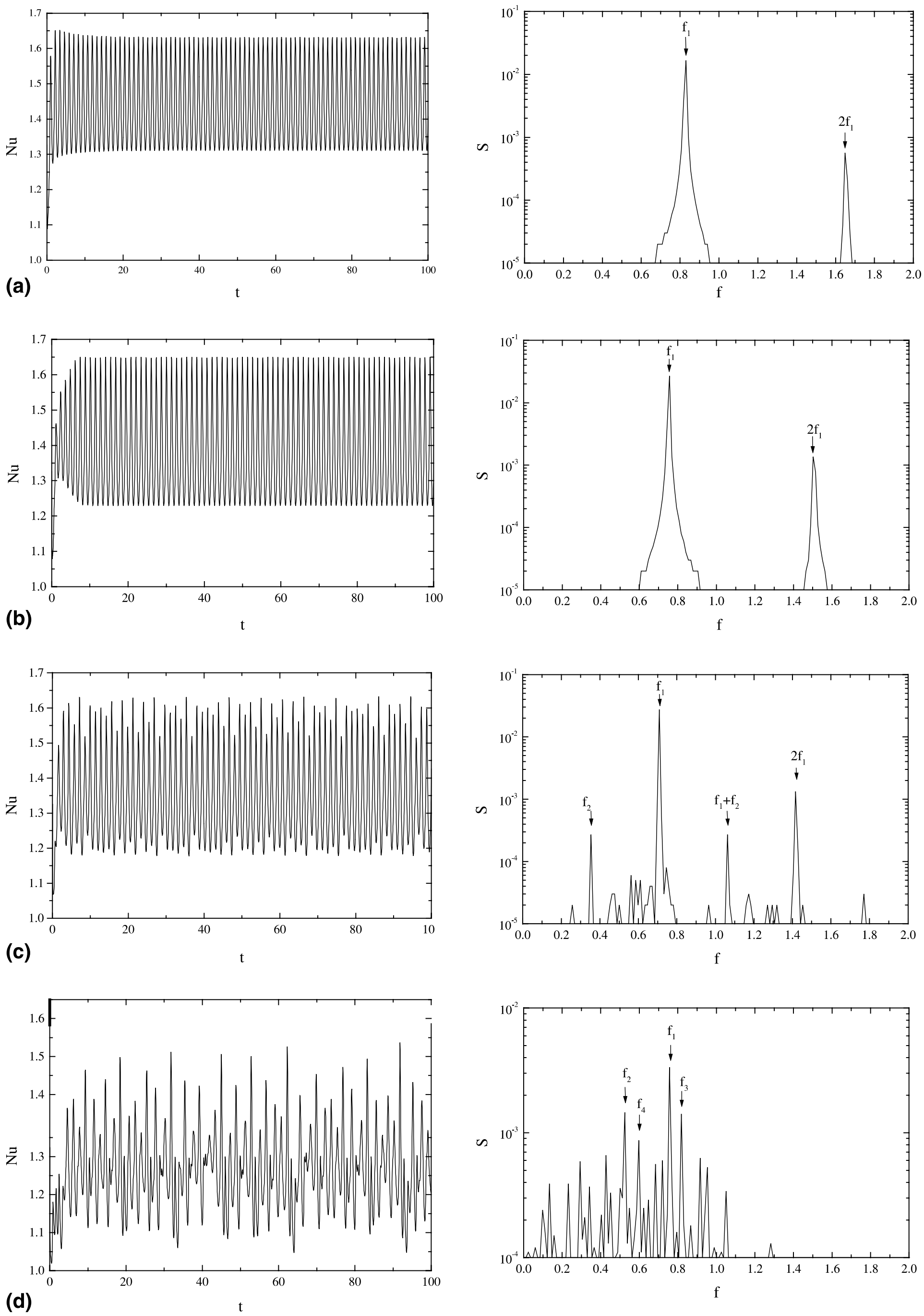

Fig. 2. Oscillations of Nu and their power spectra obtained in the numerical calculations for various N at $R_c = 50$ and $Le = 10$. f_n shows fundamental frequency peaks. (a) $N = 0.85$; (b) $N = 0.80$; (c) $N = 0.75$; (d) $N = 0.65$.

tional results were obtained for other aspect ratios. In order to eliminate the first transient effect, the calculations with long enough time, $t = 100$, were performed.

Oscillatory convection is found in the range between two critical values of the buoyancy ratio N, $N_{\min}$ and $N_{\max}$. This N is expected to have a significant effect on the characterization of the oscillation. Fig. 2 shows the time-development of the oscillations of *Nu* for various N at $R_c = 50$ and $Le = 10$ and their power spectra after the oscillation is fully developed. FFT was applied to a set of 2048 points in time($18.12 \leq t \leq 100$). The oscillation range is $0.64 \leq N \leq 0.86$ under these conditions. When N is equal to 0.85 or 0.80, only a fundamental peak and its harmonics are observed. The fundamental peak for the smaller N shifts to the lower frequency. At $N = 0.75$, two fundamental peaks, their combinations, and some other peaks are observed. When the buoyancy ratio N approaches $N_{\min}$ ($N = 0.65$), the convection becomes extremely complex with growing incommensurate peaks.

The oscillation takes place not only with the parameters explained in the above but also with other combinations of N, Le and R_c. Fig. 3 shows the oscillating region of the convection in maps of N vs. Le for various R_c. $N_{\min}$ and $N_{\max}$ are determined within error of $|\Delta N| = 0.01$ by the numerical calculations. The maps are shown only for $Le \geq 1$ because almost all fluids have the property of $Le > 1$ and that the governing equations are symmetrical for θ and ϕ. While the oscillatory convection does not take place at $Le = 1$ even for the large R_c (< 300) as indicated in the map, the oscillation is found for small R_c (≥ 50) at $Le \geq 2$. The oscillation range of N is very small when Le is close to 1, because the system may become similar to that having the single diffusive component. The oscillatory convection does not take place at $Le = 1$ even for the large R_c (< 300) as indicated in the map. The oscillation range becomes large as Le increases, and it becomes saturated around $Le = 10$–20 in any R_c. Since the condition of large Le and R_c means that the system is governed almost only by the single diffusive component, the oscillation is also diminished. Oscillating convection is observed even at N equal to 1.0 in the present calculation when R_c is larger than 200, though $Nu = 1$ is the exact solution from the analytical solution at $N = 1$ [7,8]. Since this solution is on a critical balance of the diffusion of heat and mass, the oscillation may take place in the numerical calcula-

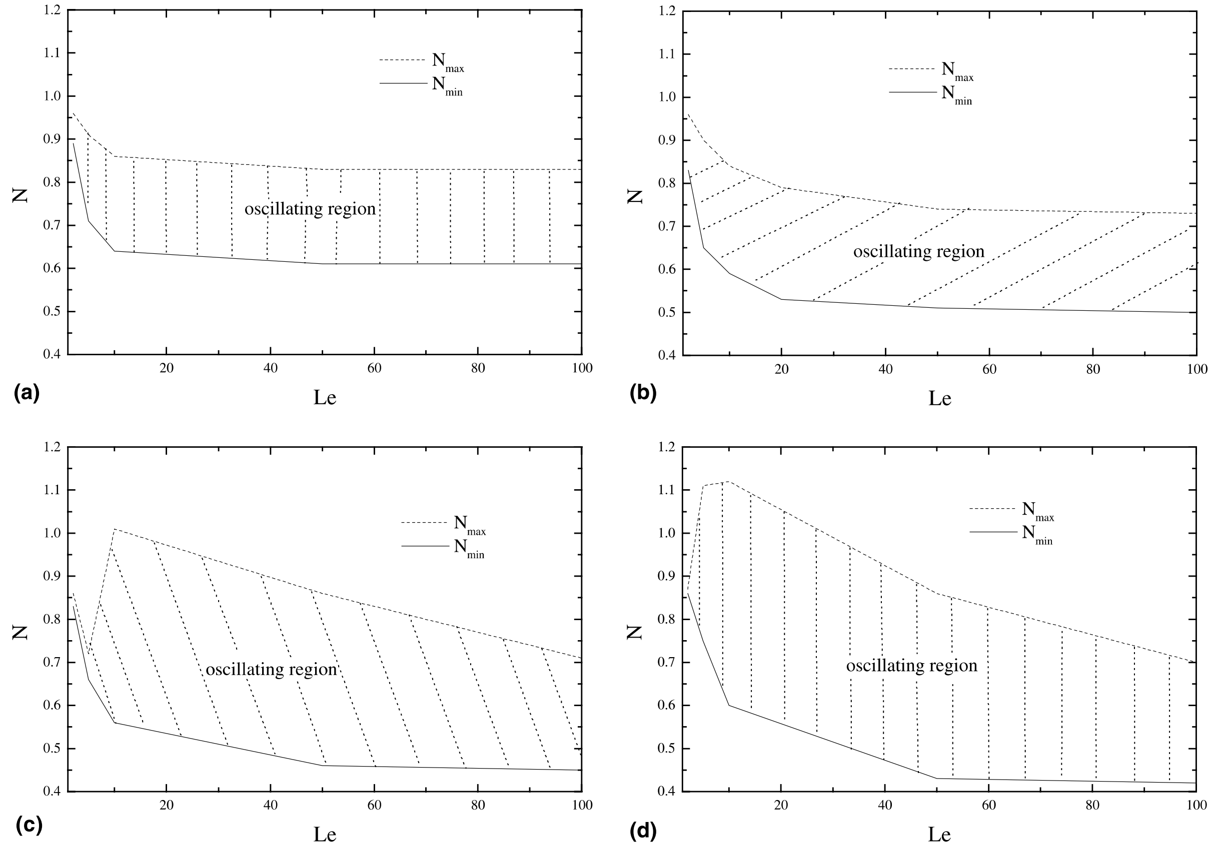

Fig. 3. The parameter range in N–Le map to give the oscillating solutions in the numerical calculations at: (a) $R_c = 50$; (b) $R_c = 100$; (c) $R_c = 200$; (d) $R_c = 300$.

tion. Outside the oscillation range, the analytical solution shown in [8] can be used to evaluate *Nu* and *Sh*.

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